

BUILDING A MODEL FOR SCORING 20 OR MORE RUNS IN A BASEBALL GAME

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How often can we expect a Major League Baseball team to score at least 20 runs in a single game? Considered a rare event in baseball, the outcome of scoring at least 20 runs in a game has occurred 224 times during regular season games since 1901 in the American and National Leagues. Each outcome is modeled as a Poisson process; the time of occurrence of one of these events does not affect the next future occurrence. Using various distributions, probabilities of events are generated, goodness-of-fit tests are conducted, and predictions of future events are offered. The statistical package R is employed for analysis.

Introduction. In their home opening weekend of the 2009 season, the New York Yankees and their fans were excited about the future of the Bronx Bombers. The new “cathedral” in Major League Baseball had cost 1.5 billion dollars to construct. There were certain to be sold-out crowds. The Yankees had made several off-season signings. Management and fans had expectations of great pitching and a high-powered offense, capable of scoring LOTS of runs. That’s what Yankees fans wanted in the new ballpark and that’s what they saw; unfortunately, it was the Cleveland Indians who provided the power. On April 18, 2009, the Tribe scored 14 runs in the second inning en route to a 22–4 victory over the New York Yankees. Although teams have scored 20 runs in a game before, this game established a record for the most runs scored in a 2nd inning (14), surpassing a record previously held by the Yankees.

This led to a curious question: How often does a Major League team score 20 runs in a game? Can the historical occurrences be used to predict when the next one might occur? Can the event of scoring twenty or more runs in a game be classified as a rare event?

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Some scoring history. In 1901, the American League joined the National League as the second official league in professional baseball. In that inaugural year for the new Major Leagues, eight games saw a team score twenty or more runs. The Boston Americans beat the Philadelphia Athletics 23–12, in a game on May 2, 1901, in the first occurrence of this feat. The Brooklyn Superbas accomplished the event twice in 1901. Ninety-eight years later, in 1999, the feat happened a record nine times.

On August 25, 1922, the Philadelphia Phillies played Chicago at Cubs Park (later, Wrigley Field). Chicago scored 10 runs in the bottom of the 2nd inning and 14 more runs in the bottom of the 4th. By the end of the 4th inning, the Cubs had a commanding 25–6 lead. The Phillies didn't give up, fighting back to score 17 runs over the final five frames, but it wasn't enough. Final score: Chicago 26, Philadelphia 23. This was the first time each team would score over 20 runs in a game. Chicago didn't need to bat in the bottom of the 9th, but the two teams combined for 98 at-bats, 21 walks (for a total of 119 plate appearances), 12 doubles, 2 triples, and 3 home runs. And this was the Dead Ball Era. There were 49 runs, 51 total hits, 9 errors committed, and a combination of 25 runners left on base. Tiny Osborne, a rookie pitcher with the Cubs, picked up his second save. Cubs leadoff hitter Cliff Heathcote was a perfect 5-for-5, with 2 doubles and 2 walks. He knocked in 4 runs and scored 5 himself. According to the box score, 7000 fans witnessed this slugfest, which lasted only three hours and one minute.

On June 24, 1950, Pittsburgh was playing in Brooklyn when the two teams put on a display of batting practice during the game. After 7 innings, the Dodgers were leading 14–12, but the Bums exploded for 5 runs in their half of the 8th. The game had to be suspended with 1 out in the bottom of the 8th inning with the score Dodgers 19, Pirates 12. The game was completed on August 1, and Brooklyn would add 2 more runs, running their total to 21.

On May 17, 1979, Philadelphia was again playing the Cubs, this time at Wrigley Field. After 3-1/2 innings, the Phils found themselves with a 17–6 lead (they had scored 7 runs in the top of the 1st, only to allow 6 runs in the bottom of the 1st). Both teams hammered pitching back and forth, and after 9 innings, the score was tied, at 22–22. The Phillies scored in the 10th on a Mike Schmidt solo home run. In the four-hour contest, Schmidt had blasted two home runs, and his Cubs slugging counterpart, Dave Kingman, hit three. There were 109 at-bats plus 15 walks, breaking their old record of plate appearances in a game (with one batter hit by the pitch and two sacrifice flies, 127 men came to the plate that day). There were 10 doubles, 3 triples, and 11 home runs. Each team committed two errors. The Phillies avenged the 1922 loss to the Cubs, as this is only the second game in which two teams each scored more than 20 runs.

The Cleveland Indians started the 1925 season by beating St. Louis, 21 to 14, to establish the most runs scored by one team (and by both teams) on Opening Day. 1925 is one of three seasons in which 20 or more runs were scored in eight different games. The others are 1901 and 1923.

Despite allowing 22 runs to the Indians in 2009, the New York Yankees have accomplished the rare feat of scoring 20 or more runs in a game the most times—25. That’s not much consolation to having it done to them early in the season in a new stadium. However, the 1939 Yankees and the 1950 Boston Red Sox are the only two teams to accomplish the feat three times in the same season. The Sox scored 20 and 29 runs, respectively, in consecutive games in June 1950. The only other consecutive games log belongs to the Pittsburgh Pirates, who scored 21 and 24 runs, respectively, in consecutive games in June 1925. The 29 runs scored by the Boston Red Sox in 1950 (and equaled by the Chicago White Sox in 1955) stood as a record until 2007, when the Texas Rangers dropped 30 runs on the Baltimore Orioles in the first game of an August 22nd double-header. Each starting Rangers player had at least two hits. Wes Littleton was awarded a save in the lopsided victory, since he pitched three innings in relief of Kason Gabbard, allowing no runs and only two hits.

Of the 30 teams in Major League Baseball (including their previous teams since 1901), only Arizona, Houston, and Tampa Bay have not scored 20 runs in a game. For example, the Washington Nationals have not accomplished the feat, but the Montreal Expos (who became the Nationals in 2005) did score 21 runs in a 1996 game.

Modeling the data as a Poisson process with the exponential distribution.

In Huber and Glen (2007), a rare baseball event is considered to be an outcome which occurs in fewer than 1% of all games played. Scoring 20 or more runs in a game falls into this category. Since 1901, when fans had sixteen teams to root for, a team has scored at least twenty runs only 224 times. For a frequency distribution by season, see Figure 1. Given that 171,797 games have been played in the regular seasons and there were 222 occasions when a team scored 20+ runs (from 1901 through the end of the 2008 season, with two events already in 2009), this amounts to a percentage of 0.13%.

With a value of 0.13%, this is indeed a rare event that can be modeled as a Poisson process. The Poisson distribution is a discrete distribution that is often employed to model the occurrence of events of a particular type over time. In addition, if the annual number of games in which at least 20 runs are scored follows the Poisson process, the exponential distribution can be used to model the times between consecutive occurrences. We will call these times the “inter-arrival times (IATs).” A random variable T is said to have an exponential distribution if its probability density function is

$$f(t; \lambda) = \begin{cases} \lambda e^{-\lambda t}, & t \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\lambda > 0$. Additionally, if T has an exponential distribution with parameter λ , then the expected value of T equals $1/\lambda$ and the variance of T is equal to $1/\lambda^2$. Both the mean and standard deviation are the same. An important distinction of the exponential distribution is its “memoryless” property. It is well known that the only continuous distribution that models a memoryless process for inter-arrival times is the exponential distribution. The time of the last occurrence of one event doesn’t affect the probability of the next occurrence, so for our data, it is reasonable to assume that these three processes or rare events are memoryless. For this data, λ has an estimated value of 0.00131 or an average IAT of 760.8 games.

In order to develop an exponential model for the data, we needed to calculate the inter-arrival times between games in which the event occurred. Using a technique developed by Huber and Glen (2007), we followed a simple algorithm. Starting with the “Play Index” research tool at www.baseball-reference.com (accessed May 2009), we generated a list of all occurrences, sorted by year. However, individual game dates were not provided. So, we went to www.retrosheet.org (accessed May 2009) and investigated the annual game logs by team. These provided us with verification that the game actually took place with a team scoring at least 20 runs, and it gave us the date.

We next counted the total number of games played in the Major Leagues at the end of the day before an event. For example, the New York Giants scored 22 runs against the Cincinnati Reds on June 5, 1912. The standings at the end of the day on June 4th (the previous day) listed 650 total games played. Since it takes two teams to play a game, we divided 650 by 2 to obtain 325. We then added 1 to that number to get 326. Often, several games started at once on a given day, which makes it somewhat impossible to ascertain the exact order of games played, especially in the early part

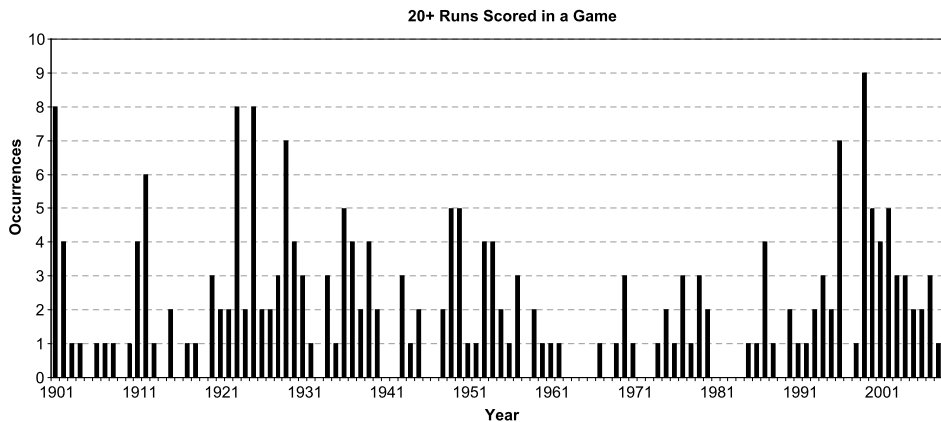


FIG. 1. Frequency distribution of 20+ run games by season.

of the 20th century. We therefore assumed that each game in which the rare event occurred was the first game played that day (thereby justifying the addition of 1 to the previous day's total). If the event occurred in the second game of a double-header, we assumed it was the last game played that day. We remained consistent with this approach for each game. Continuing with our example, the New York Giants put up 21 runs against the Boston Braves on June 20, 1912, two weeks after their first offensive barrage. This was calculated to be the 425th game of the season. By subtracting the earlier number from this one, we find that the inter-arrival time (time between rare events) was $425 - 326 = 99$ games. We used a continuous count, meaning that we wrapped data from one season to the next. The Philadelphia Athletics scored 21 runs against the Detroit Tigers on September 23, 1913, the first event since the last Giants' game. There were 1232 games played in 1912. The A's game was the 1156th of the season, giving a difference of 731 games.

Results.

A single exponential model for the data. Our preliminary modeling approach was to use all the data and fit an exponential model (assuming the memoryless property held for 20 run plus game rare events). The maximum likelihood (ML) estimate for the exponential model parameter is simply one divided by the sample average. With an average IAT of 760.8, this results in an estimate of 0.001314444 for the exponential model parameter.

We next explored how well this model fit the observed data. Initially, since this is believed to be a memoryless process, we felt that using the exponential model would be reasonable. However, we observed issues with the fit throughout much of the distribution. Figure 2 shows a comparison of the exponential model cumulative distribution function (CDF) using the estimated lambda versus the empirical distribution function (EDF) for the observed inter-arrival times. At first glance, the model roughly seems to follow the data. However, there are regions of the graph where the model does not seem to fit well, which we will investigate further.

What does the chart tell us? We would expect an 80% chance of seeing a team score twenty or more runs in a game within 1200 games played (half a season). Both curves intersect in this region. Both the EDF and CDF curves predict a probability of over 90% for a rare event occurrence within a season. What about an upcoming weekend, where all teams will play three games each? That means 45 games total, and we would expect less than a 10% chance (closer to 6%) of seeing an explosion of 20 runs by a single team in a game.

To further explore the fit of the model graphically, the best approach is to use a quantile-quantile (QQ) plot. The QQ plot in Figure 3 shows the observed quantile values on the x -axis and the theoretical quantiles for those

values using the exponential distribution on the y -axis. If the distribution fits the data well, the result should be a 45 degree line—clearly, this distribution is not a good fit. For smaller inter-arrival times, there are more observed values than one would expect based upon the exponential model, while with larger times, there are not as many observed values.

We used three standard goodness-of-fit (GOF) tests to confirm the conclusions from the graphical analysis of the exponential model. The three tests used were the Pearson chi-squared (χ^2) test (binning the data by decades, producing a test with 10 degrees of freedom), the Kolmogorov–Smirnov (K–S) test, and the Anderson–Darling (A–D) test. In all cases, the null hypothesis is that the model fits the data well. Results for these three tests are shown in Table 1.

The results for all three tests strongly reject the null hypothesis that the exponential model fits the data. For the χ^2 test, the critical value for the test at a 5% significance level is 18.31 and the test statistic value is well above this threshold. For the A–D test, at an even more stringent significance level of 0.25% the critical value is 2.534; the test statistic again exceeds this value by a large amount. The K–S statistic exact p -value is 0.000026—

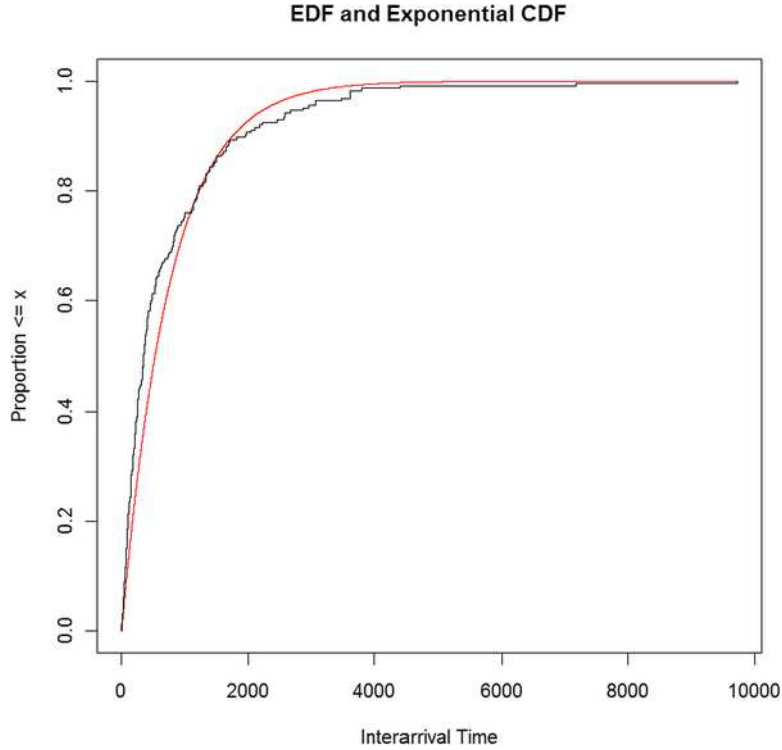
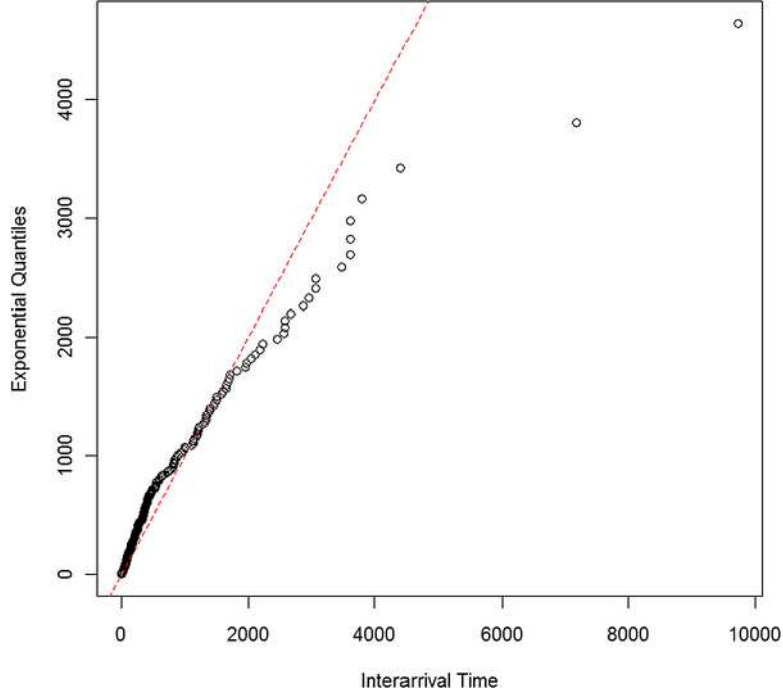


FIG. 2. Observed IAT data (EDF) and the fitted exponential model (CDF).

FIG. 3. *Quantile-quantile (QQ) plot.*TABLE 1
Goodness-of-fit tests for single exponential model

GOF test	Test statistic	p -value
K-S test	0.1581	$\ll 0.0001$
A-D test	8.207	$\ll 0.0025$
χ^2 test	44.616	$\ll 0.05$

meaning that there is little probability that the observed data follows the exponential model we fit. The GOF tests tend to reject more often for large data (they have great power to detect departures from the model in such cases). However, the tests reject very strongly in this case and the graphical analysis suggests problems with the model as well. We must conclude that the single exponential model is not a good choice for 20 plus run games.

An improved exponential model based on Era. To determine the best approach for improving the model, we first considered the data points with the largest residual (“worst fit”) values. Due to the nature of the data, these cases are almost certain to be those with the longest IAT values—the top

TABLE 2
10 worst fit data points using single exponential model

Year	Team	Games since previous 20+ run event
1985	Philadelphia Phillies	9723
1967	Chicago Cubs	7181
1974	Kansas City Royals	4408
1975	Boston Red Sox	3608
1992	Milwaukee Brewers	3471
1969	Oakland A's	3612
1910	Boston Doves	3067
1948	St. Louis Cardinals	3620
1986	Boston Red Sox	2960
1990	San Francisco Giants	3795

ten such points shown in Table 2 reflect this fact. However, these games are interesting as they fall largely (8 of the 10) into two time frames: 1967–1975 and 1985–1992. Looking at the data more closely, we observe that there were no 20 run games from 1963–1966 and 1981–1984, and generally fewer such games in the years these 8 data points fall into.

The outlier analysis above suggested that the exponential model may not be appropriate because the rate of 20 plus games is not constant for the entire time period of the data. There appear to be periods when the rate is slower and there are likely other periods where the rate increases. Scoring and offensive production has changed periodically during baseball history, leading to the identification of Baseball Eras [taken from www.netshrine.com (accessed May 2009)]. These eras are typically defined in the six periods in Table 3.

The years identified in the outlier analysis fall very nicely into the eras of Expansion and Free Agency. It thus seemed very reasonable to believe that an appropriate model would use the exponential distribution but with a different estimated rate parameter for each Baseball Era. Looking at the

TABLE 3
Baseball Eras

Era	Dates
Dead Ball	1901–1919
Lively Ball	1920–1941
Integration	1942–1960
Expansion	1961–1976
Free Agency	1977–1993
Long Ball	1994–current

TABLE 4
IAT data by Baseball Era

Era	20+ games in Era	Mean IAT	SD IAT
Dead Ball	33	612.5152	859.7318
Lively Ball	68	400.7941	439.4383
Integration	37	650.1351	813.2032
Expansion	12	2307.3333	2088.3358
Free Agency	22	1561.4091	2135.8498
Long Ball	53	709.7547	709.7914

data by Baseball Era (Table 4) certainly supports this approach. We see a noticeable drop in the rate of 20+ games in the Free Agency and Expansion Eras, as expected. It does come as a small surprise that 20+ games in the “Dead Ball” Era do not occur at a slower rate; in fact, the average IAT for the era is one of the shortest. There is also reasonable evidence that exponential models might be appropriate within eras since the means and standard deviations are often close (a feature of the exponential distribution). The “Dead Ball” Era has the largest difference in mean and standard deviation. We will further discuss this era later.

To confirm the observed differences in eras statistically, we performed an Analysis of Variance (ANOVA) on the IAT by era. The ANOVA was statistically significant ($p < 0.0001$) so at least two eras differ significantly statistically. The pair-wise comparisons (depicted graphically using the family-wise 95% confidence intervals in Figure 4) confirm what we observed—the two eras in question (Expansion and Free Agency) differ from all other eras significantly. They do not, however, statistically differ from each other, nor do any of the other eras.

Fitting exponential models to each Baseball Era, in similar fashion to the previous modeling efforts, resulted in the estimated rate parameters in Table 5. A graphical depiction of the resulting models are shown for each of the six Baseball Eras in Figure 5.

The graphs of the models fit by era dramatically point out the previously identified eras (Expansion and Free Agency) which have lower rates of 20 run games. The other eras are more similar, with the exception of the “Lively Ball” Era in which high scoring games occur at a slightly higher rate.

We assess the separate model fits in a similar fashion as we did for the single exponential model. Graphically, the QQ plots (Figure 6) and the EDF/CDF plots (Figure 7) suggest that modeling the eras separately has improved the fit significantly. There are some possible departures from the model suggested by the QQ plots for a few eras. However, note that most of the data points are within the 95% confidence bounds shown on the graphs. In general, these bounds are much wider for the larger IAT due to fewer

TABLE 5
*ML estimates for exponential rate
 parameters in each Baseball Era*

Era	Rate parameter
Dead Ball	0.001632613
Lively Ball	0.002495138
Integration	0.001538142
Expansion	0.000433401
Free Agency	0.000640466
Long Ball	0.001408975

data points—this is also the area where we would expect more variability and possibly outliers. The more interesting portions of the plot are those eras where there appear to be more than one “line” of points. The “Dead Ball” Era is an example. At lower IAT values in this era the plotted QQ

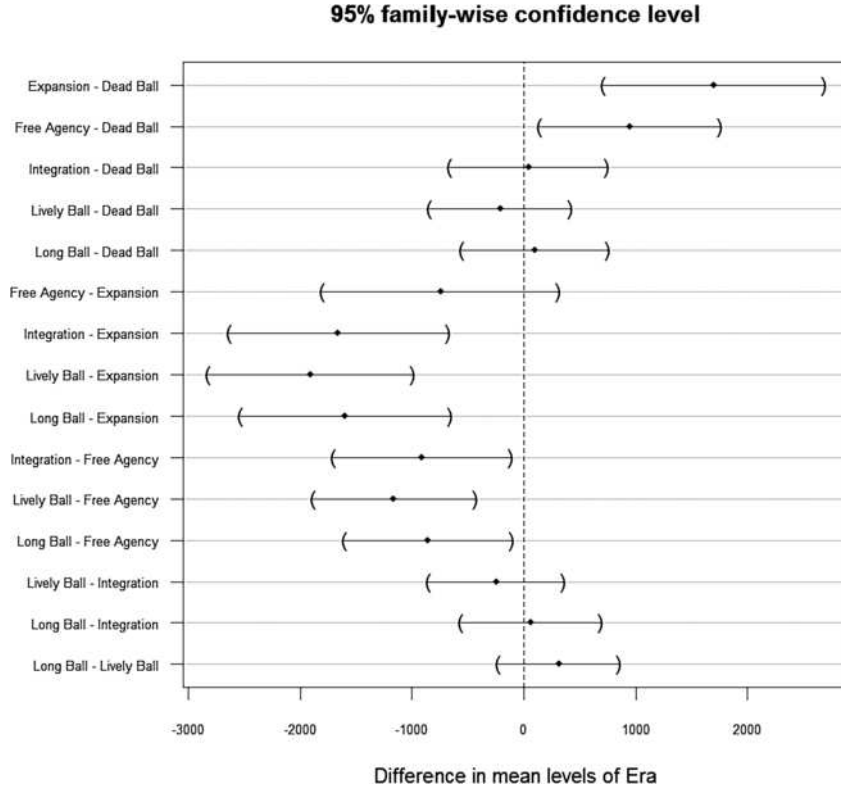


FIG. 4. *Family-wise confidence intervals for pair-wise comparison of Baseball Eras.*

points have a lower slope than the points for higher IAT's. This suggests the possibility of a missing “factor” in the model.

The fitted model CDF's all appear reasonable fits to the observed EDF's except for the “Dead Ball” Era where the model first underestimates the rate then overestimates.

GOF test results for the six eras are shown in Table 6. The K-S test rejects the null hypothesis of model fit only in the “Dead Ball” Era. Similarly, using exponential tables for the A-D test statistics [D’Agostino and Stephens (1986)], only the “Dead Ball” Era model fit is questionable. The GOF test results confirm what we observed graphically—we appear to have well fit models in all but the “Dead Ball” Era.

The modeling issues in the “Dead Ball” Era are the result of inconsistency in certain years during the period. In most years, the rate of 20+ run games is very low in this era, but in 1901–1902 and then again in 1911–1912 there are an astonishing number of such games. In 1911 there are 4 twenty run games and in 1912 another 6 for a total of ten such games in two years. The 1901–1902 seasons have even more high scoring games recorded with 12 (8 in 1901 alone). This is a very high rate of these rare events for any

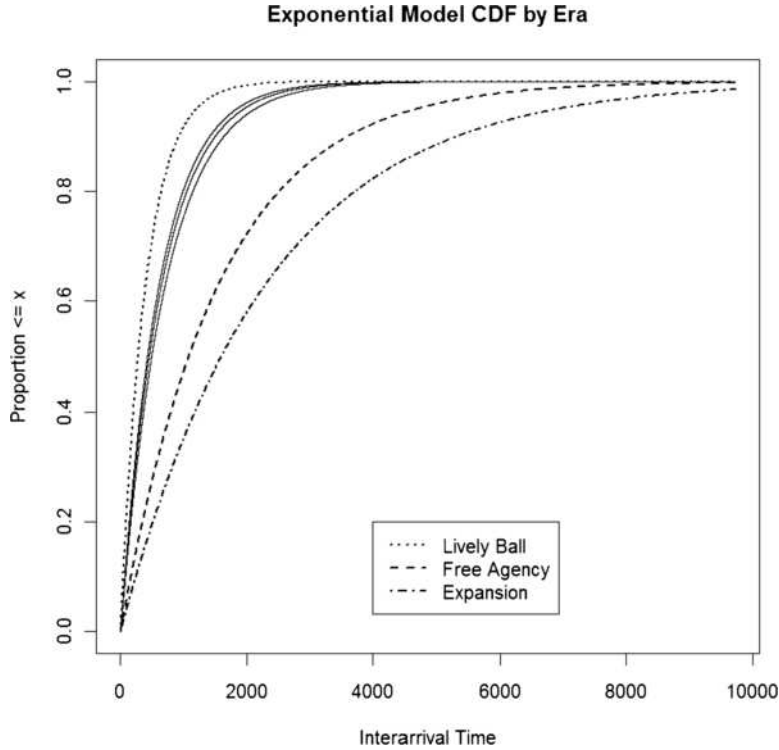
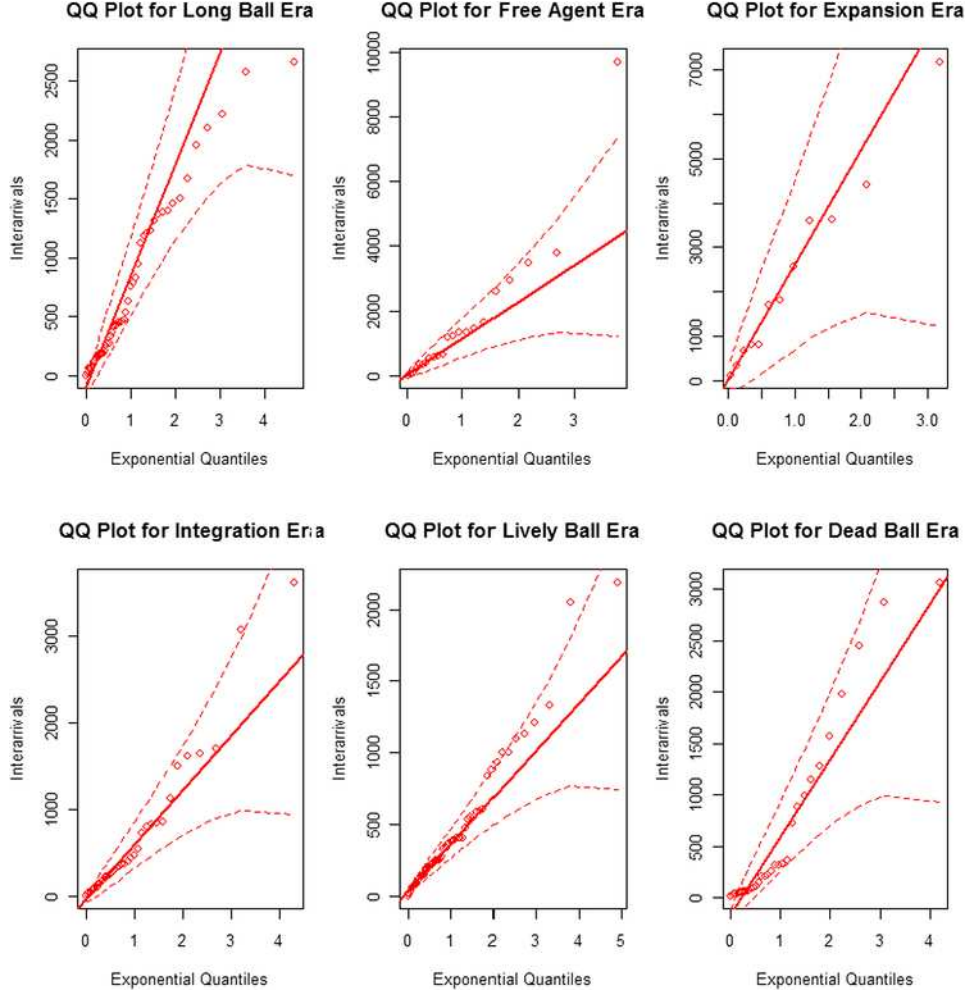


FIG. 5. *Exponential models by Era.*

FIG. 6. *QQ plots for exponential models fit to each Baseball Era.*TABLE 6
GOF tests for Baseball Era exponential model fits

Era	A-D statistic	A-D approx.	
		p -value	K-S p -value
Dead Ball	4.054	$\ll 0.0025$	0.03348
Lively Ball	0.6172	> 0.25	0.5375
Integration	0.8282	0.15–0.20	0.5332
Expansion	0.8385	0.15–0.20	0.9817
Free Agency	0.7036	> 0.25	0.5431
Long Ball	0.728	> 0.25	0.5585

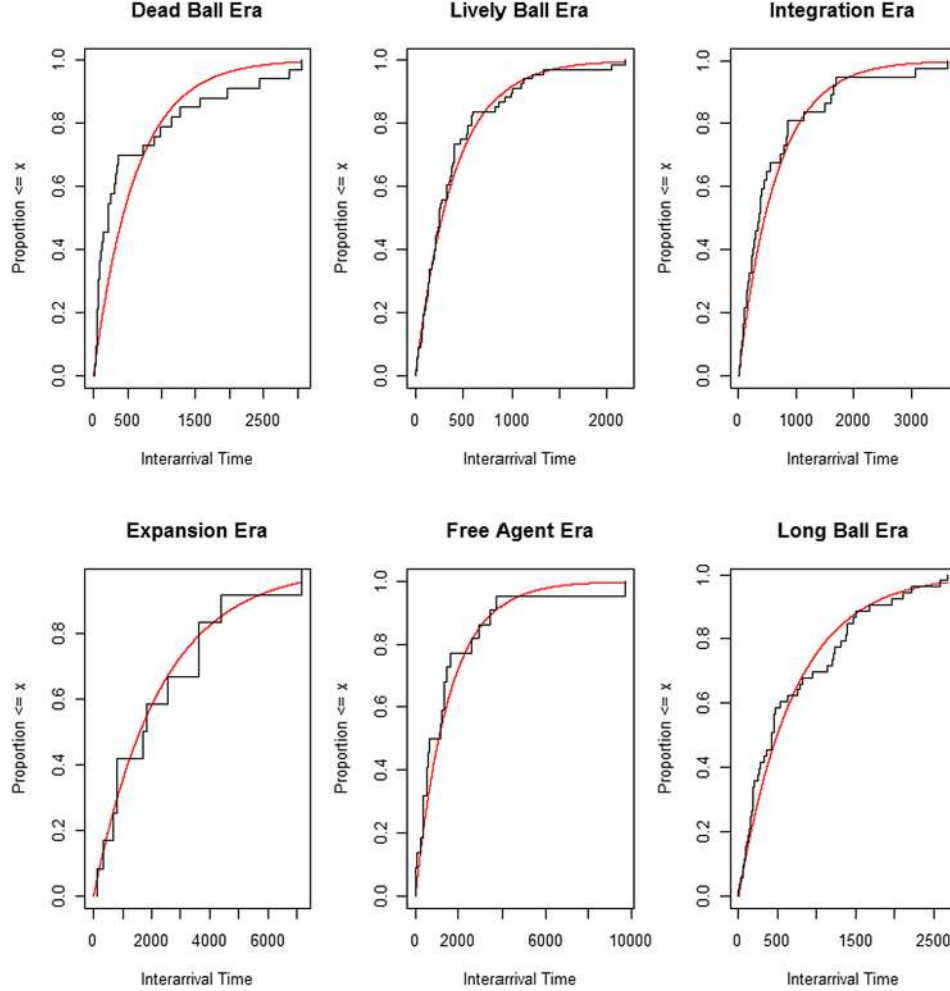


FIG. 7. *EDF and fitted exponential model CDF for each Baseball Era.*

era, and particularly surprising in the “Dead Ball” Era where there are a total of 11 other 20+ run games in the remaining 16 years of the era. The increase in 1901–1902 could have been the result of expansion (which often has the effect of “watering” down pitching), as the new American League doubled the number of teams from 8 to 16. The higher rates in these four years creates the apparent “dichotomy” we observed in the graphs of the model fit and explains the failure of the exponential model for this era.

Conclusion. We model the rare event of games with a team scoring 20 or more runs as a Poisson process using the exponential distribution to model the IAT between such games. Our work suggests that the rate of 20+ run

games is not constant over time but, rather, the rate varies in Baseball Eras. Separate models fit to each Baseball Era are successful for eras after 1920. In the “Dead Ball” Era prior to 1920 there are issues with the fitted model due to several years of anomalously high rates of the high scoring games.

The models fit after 1920 are revealing. During the “Lively Ball” Era beginning in 1920 the rate of 20+ run games is the highest of any period in baseball history, with an average of only 400 games between these events. The occurrences decrease starting with the “Integration” Era in 1942 when the average number of games between offensive explosions climbs to 650 games. This agrees with what is generally true in this era—offensive numbers declined slightly from the “Lively Ball” Era.

A further decrease in high scoring games occurs in the two eras to follow—“Expansion” and “Free Agency.” In fact, from 1961–1976 fans attending baseball games regularly could expect to wait over 2300 games between 20 run outbursts. This, again, agrees with known baseball history. The “Expansion” Era is marked by a decrease in offense as the league added teams (diluting the talent pool) and, more importantly, the strike zone was enlarged, giving pitchers a tremendous advantage. The drop in offense led to some changes in the latter part of the “Expansion” Era—fans demanded more scoring. In 1969 the pitching mound was lowered and in 1973 the American League adopted the designated hitter. This does raise the rate of 20+ games in the “Free Agency” Era, although not back to the rates of previous eras. It is not until the “Long Ball” period starting in 1994 that the occurrences of 20+ run games increase to levels of the eras before 1960. Somewhat surprisingly, the rate is still not quite as high as it was then, despite the tremendous increase in home runs observed in the last fifteen years.

How can we use this data? Let’s explore the probability of predicting a next occurrence. The Philadelphia Phillies scored 20 runs on June 13, 2008, the second time the World Champions scored 20 runs in that season. This June game proved to be the last time twenty runs would be scored by one team in the 2008 campaign. This means that the first game on Opening Day, 2009, would have an IAT of 1425. According to Figure 7 and the Long Ball Era, an IAT of 1425 would lead to an 87% chance of an occurrence of a team scoring 20+ runs in a game. Therefore, the Cleveland–New York game on April 18, 2009, had a high probability of being a game in which a team scored 20 or more runs. Unfortunately for Yankees fans, it was the Indians who accomplished the feat. If a month goes by (approximately 300 games), there would be a 34.3% chance that another event would occur. This probability increases to 57.4% after 600 games, given the rate parameter used.

REFERENCES

- HUBER, M. and GLEN, A. (2007). Modeling rare baseball events—Are they memoryless?
Journal of Statistics Education **15**.
- D'AGOSTINO, R. and STEPHENS, M. (1986). *Goodness-of-Fit Techniques*. Dekker, New York. [MR0874534](#)

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